

Selected (Partial) Answers

- 12 If the budget (size) runs out when the next item is too expensive, the consumer will jump to another item that has lower MU per dollar, with a lower price tag. But Neyman-Pearson randomizes on whether to include this point in the rejection region: it does not skip to another sample point with a smaller likelihood ratio. And the consumer problem can be solved this way as well, with the interpretation that the consumer offers a suitable bet to the seller.
- 19 [Elena] the last part of the question is apparently misspecified: the firm chooses fixed capacity for all values of a. The problem should be rewritten to deal with this.
28. [x1e203.90 Q1]
 Quasiconvexity can be stated as $F(\mathbf{x}) \leq \max [F(\mathbf{x}'), F(\mathbf{x}'')]$, where \mathbf{x} is a convex combination of \mathbf{x}' and \mathbf{x}'' .
Proof of Quasiconvexity of $v(p,y)$ in p .
 Let \mathbf{p} and \mathbf{p}' be two price vectors, and let $\mathbf{p}'' = t\mathbf{p} + (1-t)\mathbf{p}'$ be a convex combination of these, with $0 \leq t \leq 1$.
 $v'' = v(\mathbf{p}'', y) = u(\mathbf{x}'')$ for some \mathbf{x}'' with $\mathbf{p}'' \cdot \mathbf{x}'' \leq y$. Say $\mathbf{p}'' \cdot \mathbf{x}'' \leq y$. Then $v'' \leq \max(v, v')$, since either \mathbf{x}'' or \mathbf{x} (perhaps both) must be available at the price vector \mathbf{p}'' .
 NOTE: this does not require that u is quasiconcave (although it does require that a maximum exists).
 Let \mathbf{p}'' be Houston, \mathbf{p} is LA and \mathbf{p}' is Miami. Then Houston can't be strictly better than both LA and Miami.
29. a. Just like an inferior good in consumer theory.
- b. One answer is to say why not? None of the properties of a profit function is violated if $\partial^2 \pi / \partial p \partial w$ is positive. The Hessian of the profit function must be positive semi-definite, so each entry on the diagonal must be non-negative. If the off-diagonal elements are too large, the determinant of one of the principal minors will be negative, which is not allowed. But the off-diagonal elements can certainly be negative.

Note that linear homogeneity implies the Hessian must be singular. [?] In any case, linear homogeneity is an additional restriction, but this can easily be satisfied.

A deeper answer is that $\partial q / \partial w_i$ is positive whenever x_i is an inferior factor. The conditional demand function in this case is negatively related to output, so $\partial x_i(\mathbf{w}, y) / \partial y = \partial^2 c(\mathbf{w}, y) / \partial w_i \partial y$ is negative, meaning that MC decreases when w_i increases, by Young's theorem. So when w_i increases output rises and x_i falls, because this is an inferior input.

More formally, the factor demand function satisfies $x_i^*(p, w) = x_i^c(\mathbf{w}, y^*(p, w))$, where x^c is the conditional factor demand function. Differentiate this wrt p , and use Young's theorem applied to the profit function

$$x^*(p, w) = x^c(w, y^*(p, w))$$

$$\frac{\partial x^*(p, w)}{\partial p} = \frac{\partial x^c(w, y)}{\partial y} \frac{\partial y^*(p, w)}{\partial p}$$

$$\frac{\partial x^*(p, w)}{\partial p} = - \frac{\partial^2 \pi(p, w)}{\partial p \partial w} = - \frac{\partial^2 \pi(p, w)}{\partial w \partial p} = - \frac{\partial y^*(p, w)}{\partial w}$$

Then since y^* is increasing in p , the derivative of output with respect to the wage and the derivative of the conditional labor demand function with respect to output have opposite signs.

- c. The argument fails because there are two exceptional cases which exactly offset each other. It can happen that an increase in output reduces the use of some factor. And it can happen that an increase in

some factor price increases the optimal level of output. The point is that these two exceptional cases occur together. So in the usual case where an increase in the factor price reduces output, the factor cannot be inferior, and the Giffen exception is impossible.

A simpler but less informative answer is that Hotelling's Lemma rules out the Giffen exception.

32. [x1e203.90 Q2]

- d. If $\Delta y = x \Delta p$, the response is a pure SE. So consumption of gas must fall. The income effect is irrelevant.
- e. [Deaton-Muellbauer] This is the expenditure function for an augmented LES, with a required consumption level of 5 for good 2. The utility function can be derived by taking derivatives to get the Hicksian demands, $h_1 = \frac{1}{2}[p_2/p_1]^{1/2}u$ and $h_2 = 5 + \frac{1}{2}[p_1/p_2]^{1/2}u$; eliminate the price ratio and rescale utility to get $h_2 = 5 + [1/h_1]u$, and then write utility as $u = x_1(x_2 - 5)$.
- f. Concavity of $e(p, u)$ in p requires only that the optimal solution exists, not that the utility function is quasi-concave. In this example, the indifference curves are quarter-circles, and the optimal choice is $x_1 = 0$ if $p_1 > p_2$, $x_2 = 0$ if $p_2 > p_1$, so the expenditure function is $e = (\sqrt{u})\min[p_1, p_2]$. This is piecewise linear, so the Hessian is zero. In any case, the standard proof of concavity goes through unchanged: The expenditure function is concave in p . If p is an average of p' and p'' , let x be some plan which gives $U(x) = u$, with $p \cdot x = e(p, u)$. Then $e(p', u) \leq p' \cdot x$, since utility u can be bought for $p' \cdot x$ at prices p' ; and similarly $e(p'', u) \leq p'' \cdot x$. So $ae' + (1-a)e'' \leq p \cdot x = e$.

38. The monopoly firm cannot make positive profit if each customer pays the same price, but the consumer surplus if each customer paid marginal cost would be more than enough to allow positive profits. The problem is basically a public goods problem: if each consumer would report their demand price truthfully, then the high-valuation types could be charged more. Perhaps the town (or the monopolist) could experiment with price discrimination schemes which use some observable characteristics like income or location or family size or housing assessed values to achieve enough sorting to make the project viable.

demand is $Q = 100 - P$

MR is $100 - 2Q = MC = 20$, so π -max means $Q = 40$, $P = 60$

but then $R = 2400$, $C = 2000 + 800$ and the firm loses money.

consumer surplus is maximal when $P = MC = 20$, $CS = \frac{1}{2}80 \cdot 80 = 3200$, which exceeds FC

41. The indifference curves are convex to the origin, so corner solutions are appropriate.

The transformation curve for the economy is $M^2 + R^2 = L_M + L_R = N$

The "natural" equilibrium is one where the labor force is divided equally between the two products, and half of the consumers eat only rice, and the other half eat only meat. The relative price is 1, so the consumers are in equilibrium. The wage is determined as the marginal product when $N/2$ workers are employed, and then firms are maximizing profits.

$MPL = 1/(2M) = 1/(2R)$.

$\pi_M = M - wL_M = M - [1/(2M)](N/2)$

42. The Slutsky matrix S is a 2×2 matrix which is the Hessian of the expenditure function, and therefore symmetric. Also, S is the Jacobian of the Hicksian demand functions, and these are linear homogeneous.

$h_1(\lambda p_1, \lambda p_2, u) = h_1(p_1, p_2, u)$ $h_2(\lambda p_1, \lambda p_2, u) = h_2(p_1, p_2, u)$

Derivative wrt λ gives

$p_1 h_{11} + p_2 h_{12} = 0$ $p_1 h_{21} + p_2 h_{22} = 0$

The Slutsky matrix is just $[h_{ij}]$, which starts with four unknowns. Symmetry gives one equation, linearity gives two more, so there are three equations in four unknowns. If one of the elements of the Slutsky matrix is given, it should be possible to infer the other three, unless there is linear dependence in the equations. As long as both prices are positive, there is no difficulty in solving the equations.

Note that the elements of the matrix are evaluated at given values of p_1, p_2, u .

Also, the prices must be **known**.

43. First assume that housing itself has price elasticity less than one, so that so that the SF consumer does indeed spend more money on housing.

Fix a level of housing consumption, and draw the indifference map for the rest of the consumption bundle. Change the housing level, and look to see whether the indifference map changes. If it does not, then housing is **weakly separable** from other goods, and the effect of increased expenditure on housing is just the same as the effect of a decreased income, as far as non-housing goods are concerned. But if the change in housing consumption changes preferences over other goods, the result is not like an income effect. For example, it is easy to think of things which are complementary to housing, and things which are substitutes, so that separability fails.

- 44.. Wages must equalize utility in the two places, because the income each person gets from renting out land does not depend on where the person lives. So if A has the amenity, it must have lower wages so that people are equally attracted to both places. This is true whether the amenity affects production costs or not.

If the amenity does not affect production costs, there are more workers in A per acre of land used, so the marginal product of land is higher than in B, and land rentals are higher in A. But if the amenity directly raises the cost of production, the wage could be lower in A even if there are fewer workers in A than in B. And land rentals could also be lower in A.

In any case, a worker moving from B to A gives up some consumption to get the amenity. If the amenity is health, the wage differential measures the amount of consumption which a worker will give up in order to live in a healthy place, so it is true that the wage differential measures the extent to which people value their health. This assumes that land is used only in production, not in consumption. But if land is used in consumption the value of health must be measured by comparing both the wage differential and the differential in housing prices.

- 45.. The cost function should be increasing in w_1 and w_2 and in y , linear homogeneous in W , and concave in W . Note that these are properties of C , not of $c = \log(C)$.

Linear homogeneity of C means that if λ is added to w_1 and w_2 then λ is also added to c , and this must hold for any values of w, y and λ .

$$c(w+\lambda, y) = \lambda + c(w, y)$$

derivative of this is $c_1(w+\lambda, y) + c_2(w+\lambda, y) = 1$, with $\lambda=0$ for example

$$c_1 = a_1 + 2b_{11}w_1 + b_{01}y + b_{12}w_2$$

$$c_2 = a_2 + 2b_{22}w_2 + b_{01}y + b_{12}w_1$$

so

$$a_1 + 2b_{11}w_1 + b_{01}y + b_{12}w_1 + a_2 + 2b_{22}w_2 + b_{01}y + b_{12}w_2 = 1$$

This is an identity in w, y , so differentiate wrt each argument on both sides

$$a_1 + a_2 = 1; b_{12} = -2b_{11} = -2b_{22}; b_{01} + b_{02} = 0$$

$$c = a_0y + a_1(w_1 - w_2) + w_2 + b_{00}y^2 + b_{01}(w_1 - w_2)y + \frac{1}{2}b_{12}(w_1 - w_2)^2$$

If b_{01} is zero, then the cost-minimizing production plan has a capital-labor ratio which does not depend on y : the mrts is independent of y , so the production function is homothetic.

- 48.. **Put b equal to marginal cost, so that the consumer buys as long as the demand price exceeds c . Then put a equal to consumer surplus, which is $\frac{1}{2}(1-c)^2$. This is clearly the maximal profit choice, since the consumer is totally fleeced.**

f. The consumer won't pay two entry fees, since this would mean a total charge of $a_1 + a_2 + b_1q_1 + b_2q_2$, and the same quantity could be bought more cheaply by paying either $a_1 + b_1(q_1 + q_2)$ or $a_2 + b_2(q_1 + q_2)$, whichever is less (note that a_1 and a_2 must be nonnegative, since otherwise the consumer can make money by collecting $-a$ and buying nothing).

If the consumer is indifferent between the two firms, it must be that each firm offers the same consumer surplus:

$$\frac{1}{2}(1-b_1)^2 - a_1 = \frac{1}{2}(1-b_2)^2 - a_2 = \gamma.$$

The quantity purchased by each consumer is then $q_1 = 1 - b_1$ from 1, and $q_2 = 1 - b_2$ from 2. So profit per customer is

$$a + (b-c)q = \frac{1}{2}(1-b)^2 - \gamma + (b-c)(1-b)$$

This again gives $b = c$, and it seems that any nonnegative value of γ is consistent with Nash equilibrium. But if one firm offers a slightly higher value of γ , it will double its business, and this process drives profit to zero.

Start from the standard Bertrand equilibrium, where the equilibrium price is c and there is no fixed charge. What if one firm deviated by adjusting a and b together, so as to raise profit? Consumers buying from this firm would have less surplus, so they would refuse to buy and this deviation would not work. Conclusion: the 0-profit Bertrand equilibrium is still an equilibrium.

- 49.. Competitive equilibrium occurs when the price is between 5.50 and 6 so that the quantity supplied is 5, and the quantity demanded is also 5.

If S1 trades with B1 at 1.25, S2 with B2 at 2.25 and so on, all 10 units will be traded.

The competitive equilibrium is Pareto optimal, for the usual reasons. The decentralized equilibrium is not Pareto optimal, because the various traders faces different prices. For example B1 ends up with one unit, which he values at 1.50, while S10 sells her unit, which she values at 10.00, for some price between 10 and 10.50. So if S10 made a side deal with B1, at a price of 5 for example, then both would be better off.

- 51.. If z can't be traded, then the third agent gets $u=0$ and leaves. One and two are symmetric Cobb-Douglas types, and each consumes half of the endowment: $x_1 = \frac{1}{2}x$, $y_1 = \frac{1}{2}y$. [note that an increase in y benefits One just as much as two, in equilibrium, because the two are price-takers, so two can't just hold the new stuff off the market, and eat it].

The y and z goods must be perfect substitutes in equilibrium, and Two doesn't care about z , Three doesn't care about y , so $y_3 = z_3 = 0$. Each will consume half of his endowment, and sell the other half. One must get the other half of y (since Three doesn't want it) and the other half of z (since Two doesn't want it). So One exchanges

$$p(\frac{1}{2}x) \text{ for } q(\frac{1}{2}y) + r(\frac{1}{2}z).$$

Take $p=1$ and note that $r=q$. Then

$$q = x/(y+z)$$

Also, $x_2 = \frac{1}{2}qy = \frac{1}{2}xy/(y+z)$

$$x_3 = \frac{1}{2}qz = \frac{1}{2}xz/(y+z)$$

$$y_1 = \frac{1}{2}y, \quad z_1 = \frac{1}{2}z, \quad x_1 = \frac{1}{2}x$$

$$y_2 = \frac{1}{2}y, \quad x_2 = \frac{1}{2}xy/(y+z)$$

$$z_3 = \frac{1}{2}z, \quad x_3 = \frac{1}{2}xz/(y+z)$$

Three is better off when z is traded, Two is worse off, and One is better off, so the equilibria can't be Pareto-ranked [Two is worse off because Three supplies a good which is a perfect substitute for Two's good, according to Two's initial trading partner].

52. An expected utility maximizer will make any trade which increases the value of $\pi_A u(x_A) + \pi_B u(x_B)$, where π_A is the probability of state A. So the MRS is $[\pi_A u'(x_A)]/[\pi_B u'(x_B)]$, and if utility is linear this is the likelihood ratio. So a risk-neutral agent will trade with a risk-avertter until $x_A = x_B$, unless the risk-neutral type does not have enough of good A.

Now R believes the likelihood ratio is $\rho_A/\rho_B > \pi_A/\pi_B$. So they should trade until N's MRS is

$[\rho_A/\rho_B][u'(x_A)/u'(x_B)]$ (where u is R's utility function). This means R must be overinsured (unless there is a corner solution).

43 demand is $Q=150-P$
 MR is $150-2Q=MC=20$, so π -max means $Q=65$, $P=85$
 then $R=5525$, $C=2000+1300$ and the firm makes money.
 consumer surplus is maximal when $P=MC=20$, $CS=\frac{1}{2}130 \cdot 80=5200$, which exceeds FC

69.. The only nash equilibrium in the static game is (D,D), because this is a dominant strategy for each player, so there cannot be an optimal strategy which puts positive probability on C.

In the supergame, defection means a payoff of 1.5 now, and zero thereafter, if the other player defects forever after any defection. Playing the cooperative strategy pays $1+\delta+\delta^2+\dots=1/(1-\delta)$, and if this number is 1.5, then the cooperative strategy is an equilibrium. So $\delta=1/3$ is ok, and if δ is bigger than this, cooperation is an equilibrium, while if the players are very impatient, cooperation fails.

72.. (A_2, B_2) and (A_3, B_3) are pure Nash equilibria for the stage game.

A_1 is weakly dominated, and can thus be ignored. Once this is deleted, B_1 is weakly dominated, so the payoff matrix is 2×2 .

If A plays the mixed strategy (α_2, α_3) , B_2 pays $8\alpha_2$, and B_3 pays $7\alpha_2+10\alpha_3 = 10-3\alpha_2$. So a mixed strategy equilibrium would have to have $\alpha_2=10/11$, so that B is indifferent over B_2 and B_3 .

If B plays the mixed strategy (β_2, β_3) , A_2 pays $2\beta_2$, and A_3 pays $\beta_2+\beta_3 = 1$. So a mixed strategy equilibrium would have to have $\beta_2=1/2$, so that A is indifferent over A_2 and A_3 .

Any of these will be subgame perfect, if repeated even after a defection. Neither player can gain from a defection in the current period, so there is no point in defecting, since this will not change the future. B gains nothing by defecting from (A_1, B_1) , so the only problem is to discourage defections by A. A gains 6 by defecting now. This can be punished by reverting to the A_3, B_3 equilibrium, or to the mixed strategy equilibrium, where A gets 1. Then A gets 6 now, loses $3[\delta+\delta^2+\delta^3+\dots]=3\delta/(1-\delta)$. This means $\delta/(1-\delta)=2$ is the critical value, and so $\delta \geq 2/3$.

[seems artificial] The lowest continuation payoff from the (4,4) equilibrium for A is 1, while for B it is 4 (which B gets if no one defects). But the equilibrium (A_2, B_2) gives payoffs (2,8), thus defeating the (4,4) equilibrium.